|  |  |
| --- | --- |
| (*x* → *False*) = (¬*x*) | {¬ as →} |
| (¬(*x* ∧ *y*)) = ((¬*x*) ∨ (¬*y*)) | {∧ DeMorgan} |
| (*x* ∨ (¬*x*)) = *True* | {∨ complement} |
| (*x* ∧ (¬*x*)) = *False* | {∧ complement} |
| (¬*True*) = *False* | {¬*True*} |
| (¬*False*) = *True* | {¬*False*} |
| (*True* → *x*) = *x* | {→ identity} |
| (*x* ∧ *True*) = *x* | {∧ identity} |
| (*x* ∧ *y*) = (*y* ∧ *x*) | {∧ commutative} |
| (*x* ∧ (*y* ∧ *z*)) = ((*x* ∧ *y*) ∧ *z*) | {∧ associative} |

1. (*x* → *False*) = (¬*x*)
   1. ­­­­­­­­­­­­­­­­(¬*x*  ∨ *False*) = (¬*x*) {Implication}
   2. (¬x) *=* (¬*x*) {∨ Identity}
2. (¬(*x* ∧ *y*)) = ((¬*x*) ∨ (¬*y*))
   1. (¬(*x* ∧ *y*)) = ((¬*x*) ∨ (¬*y*))
3. (*x* ∨ (¬*x*)) = *True*
   1. (¬*x*) ∨ *x* = *True* {∨ Commutative}
   2. (¬*x*) ∨ *x* = *x* → *x* {Implication}
   3. *x* → *x* = *True* {Self-Implication}
4. (*x* ∧ (¬*x*)) = *False*
   1. (*x* ∧ (¬*x*)) = *False*
5. (¬*True*) = *False*
   1. ¬( *False* → *False*) = *False* {Self-Implication}
   2. ¬(¬*False* ∨ *False*) = *False* {Implication}
   3. ¬(¬*False*) = *False* {∨ Identity}
   4. *False* = *False* {Double-Negative}
6. (¬*False*) = *True*
   1. (¬*False*) = ¬(¬*True*) {¬*True* }
   2. (¬*False*) = *True* {Double Negation}
7. (*True* → *x*) = *x*
   1. (¬*True* ∨ *x*) = *x* {Implication}
   2. (x ∨ ¬*True*) = *x* {∨ Commutative}
   3. (x ∨ *False*) = *x* {¬*True*}
   4. *x = x* {∨ Identity}
8. (*x* ∧ *True*) = *x*
   1. (*x* ∧ (*x* ∨ *True*) = *x* {∨ Identity}
   2. (*True* ∨ *x*) ∧ *x = x* {∨ Commutative 2x}
   3. *x = x* { ∧ Absorption}
9. (*x* ∧ *y*) = (*y* ∧ *x*)
   1. ¬(¬*x)* ∧ ¬(¬*y*) = (*y* ∧ *x*) {Double Negation 2x}
   2. ¬((¬x) ∨ (¬y)) = (*y* ∧ *x*) {∨ DeMorgan}
   3. ¬((¬y) ∨ (¬x)) = (*y* ∧ *x*) {∨ Commutative}
   4. ¬(¬*y)* ∧ ¬(¬*x*) = (*y* ∧ *x*) {∨ DeMorgan}
   5. (*y* ∧ *x*) = (*y* ∧ *x*) {Double Negation 2x}
10. (*x* ∧ (*y* ∧ *z*)) = ((*x* ∧ *y*) ∧ *z*)
    1. (*x* ∧ (*y* ∧ *z*)) = ((*x* ∧ *y*) ∧ *z*)